

ORIGINAL PAPER

Effect of welded joint imperfection on the load-carrying capacity of pipe elbows subjected to in – plane bending moment

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Abstract

The defects in pipe elbows can, depending on their size and position, affect the integrity and safe service, as well as deformation ability of the piping systems in exploitation. Incompletely filled groove, which is the type of defect examined here, was observed by ultrasonic measurement on the pipeline in the regulation system of the hydro power plant Djerdap. Three-dimensional finite element analysis is performed using Simulia Abaqus software package. First, the models with dimensions of the defects observed by non-destructive examination are formed. Stress and strain fields for different loading types are shown and commented. The influence of the defect dimensions on the pipe elbow load-carrying capacity is determined through plastic collapse loads, which are obtained from bending moment - rotation angle diagrams. Twice elastic slope (TES) technique is applied. Additionally, some more severe defects are considered, in the form of sharp pre-cracks at the bottom of the defect; plastic collapse loads are also determined for these geometries. Both opening and closing bending moments are taken into consideration and the results are discussed and compared to two closed-form solutions from the literature. The influence of the boundary conditions applied for examination of the pipe elbows is commented.

Keywords: pipe elbow, incompletely filled groove, in-plane bending moment, finite element method, plastic collapse load

1. INTRODUCTION

Pressurised pipes and elbows are used in chemical, biochemical, pharmaceutical and many other industries. In the chemical industry, pipelines are typically used for the transfer of products (natural gas, crude oil, refined products) from the refinery to the customer and also to connect different parts of the plant and to transport materials within the plant. Pipelines or their segments can be exposed to a wide range of loading conditions during exploitation (internal and external pressure, bending, tension and/or compression). Pipeline regions with locally thinned wall, either due to some blunt volumetric stress concentrator, or a sharp crack-like defect, lead to the decrease of the load carrying capacity, deformation ability and fracture/fatigue resistance. Therefore, it is important to assess the influence of the wall thinning due to a defect on the pipeline structural integrity. In the literature,

there are many studies dealing with the load carrying capacity of straight pipes with different types of defects; we will mention only a few here: (Berg, Østby, Thaulow, & Skallerud 2008; Y. J. Kim, Shim, Huh, & Kim 2002; Međo et al. 2020). However, pipe elbows represent a much more complex problem for integrity assessment in comparison to the pipes, primarily due to their shape.

The pipe elbows can be the critical components in piping systems. They reduce the reaction forces and moments within the pipeline, by deforming elastically. The pipes and elbows are often joined by circumferential joints. Therefore, special attention should be devoted to the influence of possible joint defects, their detection using NDT methods and their influence on the pipeline load carrying capacity.

In the literature, different examples of failure analyses can be found: elbows without defects are considered

in Robertson, Li, and Mackenzie (2005) and Y. J. Kim and Oh (2006), while different initial defects are introduced in some other studies. These defects range from volumetric ones in (Abdel-Ghany, Ebeid, & Kasem 2014; Balan & Redekop 2005; D. S. Kim, Kim, Na, & Kim 2012; J. W. Kim, Na, & Park 2008; Y. J. Kim, Kim, Ahn, Hong, & Park 2008b; Sedmak et al. 2016) to sharp ones (pre-cracks) in Y. J. Kim et al. (2003), Hong, Kim, and Kim (2009) and Chattopadhyay, Tomar, Dutta, and Kushwaha (2004). Regarding the pre-cracks, different geometries are considered, axial/circumferential, as well as surface/through-thickness ones. The influence of the pipe and defect geometry is considered in most of the mentioned works. An interesting examination is shown by Michael, Veerappan, and Shanmugam (2012), where the influence of ovality and uneven wall thickness on the limit bending moments is discussed. A recent study by Yun, Koo, and Na (2020) deals with the application of the fuzzy neural networks in load carrying capacity prediction for elbows exposed to bending. Although the influence of the fluid stream within the pipeline is rarely taken into account in the integrity analysis of elbows, an example of such approach is shown by Chang et al. (2009), on an elbow with a volumetric defect.

The geometry of the pipe elbows considered in this work is based on the pipeline from the regulation system in the hydro power plant Djerdap. The pipeline is used for oil transport to the main turbine regulator and the entire system has the role to regulate the position and working parameters of the turbine. Nominal working pressure in this system is $p = 4$ MPa.

The main aim is to assess the influence of the welded joint defects on the load carrying capacity and integrity of the pipe elbows subjected to an in-plane bending moment (both opening and closing moments are considered). The defect shape, which is examined here, is incompletely filled groove (Fig. 1), detected by ultrasonic measurement on the internal pipeline surface.

In a way, this defect is between the sharp pre-crack and volumetric wall thinning, the two cases typically considered in the literature. Additionally, the dimensions of this defect in numerical models are varied and the effect of sharp pre-cracks is taken into account. It should be noted that the position of the defect is not symmetrical with respect to the elbow geometry, unlike the vast majority of cases from the literature.

2. MATERIAL AND METHODS

2.1. Material properties

The elbows, which are analysed in this paper, are made of steel grade 20 according to GOST 1050 standard (A53 ac-

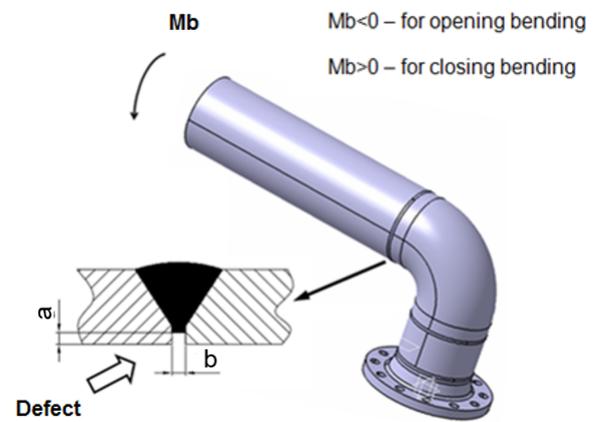


Figure 1. Pipe elbow exposed to bending moment M_b and a detail showing the joint with incompletely filled groove.

cording to ASTM); low carbon steel tubes, chemical composition in weight %: C 0.17–0.24; Si 0.17–0.37; Mn 0.35–0.65; Cu < 0.25; Ni < 0.25; As < 0.08; S < 0.04; P < 0.035. The material properties used for numerical calculations are the Young's modulus, Poisson's ratio and yield strength of 210 GPa, 0.3 and 160 MPa, respectively. In addition, the yield strength of the weld metal and the base material are very similar, so the joint is considered even-matched. External diameter of the pipe is $\varnothing 270$ mm, wall thickness is 8 mm, while the mean elbow radius is 200 mm.

2.2. Numerical analysis

In order to determine the influence of the welded joint with incompletely filled groove on the integrity of the pipe elbows subjected to the in-plane bending moment, three-dimensional finite element analysis is conducted using the Simulia Abaqus software package. Finite element analysis is based on elastic-perfectly plastic material behaviour, which is the most common approach in majority of the papers dealing with this topic (Abdel-Ghany et al. 2014; Chattopadhyay et al. 2004; Huh, Kim, & Kim 2006; D. S. Kim et al. 2012; Robertson et al. 2005). However, there are some studies where hardening is taken into account, such as J. W. Kim et al. (2008).

A model of an elbow with incompletely filled groove is shown in Fig. 2 One half of each elbow is analysed due to the symmetry and appropriate boundary conditions have been defined, which are also shown in Fig. 2. Symmetry boundary conditions are defined on the symmetry plane in all the cases. For in-plane bending, the nodes on the two ends of a pipe elbow are constrained (type of constraint: coupling). All degrees of freedom (DOF) of the nodes, on each of the elbow ends, are constrained to those of a single node, lying on the centerline. Boundary conditions on one end of each elbow are modelled as

fixed displacement and rotation in all directions. Boundary conditions on the other end of the elbows are modelled as fixed displacement in the y -direction and fixed rotation about x - and z -axes. The bending loading is applied by rotation about y -axis. Both closing- and opening-mode in-plane bending are considered (the angle of rotation is -0.2 rad and $+0.2$ rad, respectively – cca. 11.5 deg).

It should be noted that a pipe is added to the elbow geometry; its main role is to avoid excessive deformation of the elbow cross section due to the imposed rotation. In addition, it is a rather common situation in actual pipelines – they are typically connected with pipe segments. The length of the pipe used here is $4x_r$, where r is the pipeline mean diameter – in accordance with the findings of [Y. J. Kim and Oh \(2007\)](#) further increase of the pipe length does not influence the results. On the other side of the elbow model, there is no pipe attached, which will be discussed later.

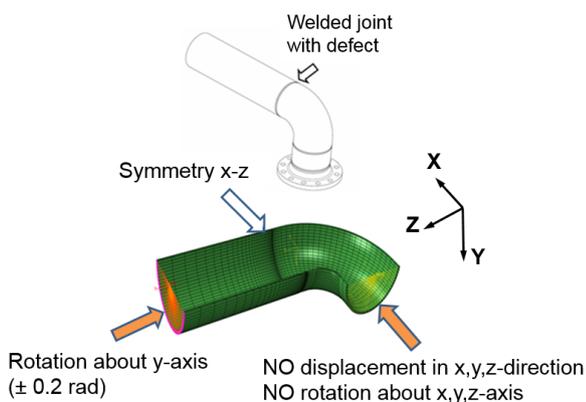


Figure 2. Half symmetry model with boundary conditions.

Finite elements, which are applied, are C3D20R (20-node quadratic brick, reduced integration), [Dassault Systemes \(2018\)](#). To improve the accuracy of the stress and strain fields, the mesh near the defect is finer in comparison to the region far away from the defect.

In this paper, models of elbows with different groove sizes ($a \times b = 2 \times 2, 2 \times 1, 1 \times 2, 1 \times 1, 2 \times 0.1$ and 1×0.1 mm; a - depth of a defect, b – width of a defect, Fig. 1) were studied. The range of defect sizes corresponds to the range detected by ultrasonic measurement. Plastic collapse moments for all the models are determined and results for 6 circumferential welding defects with different depths and widths are compared. Unlike most examples from the literature, the initial defect is not positioned in the elbow central position (extrados or intrados), but in the joint between the pipe and the elbow. In addition, one end of the pipe is constrained (fixed) rather close to the elbow, which is also rarely the case in similar analyses. This corresponds to the pronounced effect of the flange, which makes the elbow more rigid. The resistance to pres-

sure loading of these elbows was considered previously in [Dimić et al. \(2013\)](#).

Deformation of a pipe elbow under the in-plane bending moment is given in Fig. 3; undeformed and deformed meshes are shown.

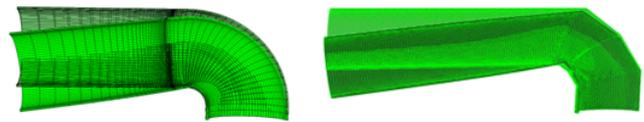


Figure 3. Deformation of a pipe elbow under the in-plane bending moment.

The plastic collapse load for the examined elbows is determined through elastic – perfectly plastic modeling of the material in the finite element analysis, as mentioned previously. An example is shown in Fig. 4, where the collapse bending moment can be determined from the moment - rotation angle curve; it represents the value obtained at the intersection of the curve and the straight line with twice the slope of the initial linear part of the curve (cca. 22.1 kNm in this example).

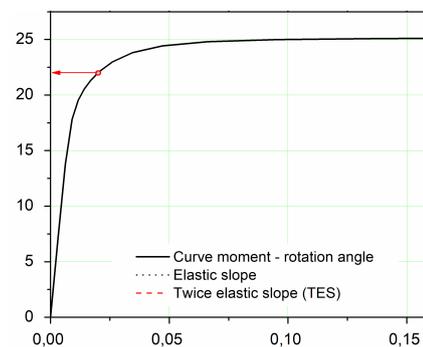


Figure 4. An example of the moment – rotation curve.

This method, namely twice elastic slope or TES, is frequently used in the literature for determination of the plastic collapse loads of pressurised components, e.g. ([Y. J. Kim, Kim, & Song 2007](#); [Y. J. Kim, Song, Kim, & Jin 2007](#); [Michael, Veerappan, & Shanmugam 2011](#); [Robertson et al. 2005](#)). Two other approaches can also be applied: TED or twice elastic displacement and TIM or tangent intersection method. Both of them are based on the same load-deformation curve as TES, but the advantage is typically given to TES due to a more subjective interpretation and more difficult application of the two latter criteria (as discussed in [Abdel-Ghany et al. \(2014\)](#)).

3. RESULTS AND DISCUSSION

3.1. Stress state – internal pressure

In the case of models loaded only by the internal pressure, the loading is distributed across the internal surface, including the surfaces of the incompletely filled groove. Such an approach is discussed in some studies and the most common approach is to apply 100% of the pressure value on the internal surface defects and 50% of this value on the through-thickness defects, (Hong et al. 2009).

The finite element mesh with the results (equivalent von Mises stress) obtained at the value of the test pressure is shown in Fig. 5. It can be seen that the stress in the straight pipe segment is much lower than the yield stress. A significant stress concentration is visible only at the internal part of the elbow (intrados) – much more pronounced than in the defect zone. This is in agreement with the conclusions from most of the literature sources that the influence of the elbow geometry is dominant in comparison to the defect geometry. However, it must be noted that some authors report a dependence on the crack geometry for some configurations of circumferentially cracked elbows, Y. J. Kim, Kim, and Song (2007).

Even in the region with the highest stress values, the equivalent stress exceeds the yield stress only up to 2%. This can also be seen based on the plastic strain field, Fig. 6, which is localized in a very small area, with the highest values approx. 0.0045%.

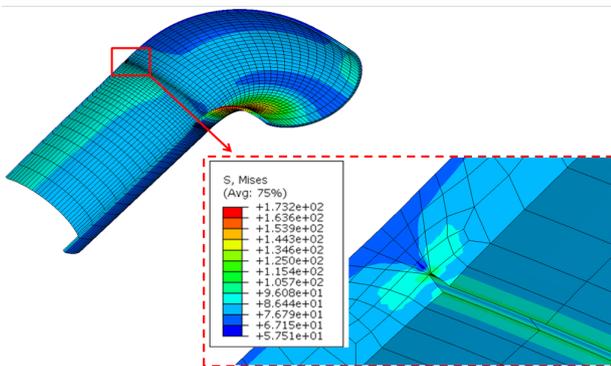


Figure 5. Equivalent von Mises stress field around the notch tip 0.1×2 mm – pressure load.

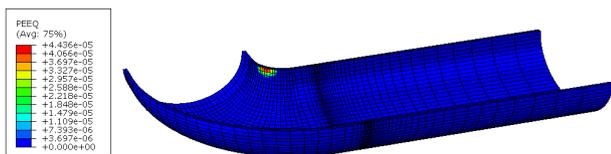


Figure 6. Equivalent plastic strain field around the notch tip 0.1×2 mm – pressure load.

An important note should be given here: the model is formed with a constant wall thickness, nominal for the

considered pipe. However, the wall thickness of the actual elbow is typically somewhat larger at the intrados – due to the production process. This makes the influence of the stress concentration on this position even less pronounced. The stress concentration in the region of the stress concentrator (notch 0.1×2 mm, i.e. the most pronounced concentrator of all the considered notches) is shown in the enlarged window in Fig. 5. As mentioned previously, the stress concentration is not pronounced. The main cause of such behavior is the fact that the internal pressure causes higher stresses in the hoop direction than in the longitudinal one; of course, the latter stresses act on the circumferential defects exposed to bending, considered later in this work.

3.2. Stress state – bending moment

Further, the influence of the bending moment on the stress fields is considered. First, the equivalent von Mises stress due to the bending (at bending angle 2°) is shown in Fig. 7. This is the distribution obtained for isolated bending load - without the internal pressure. Even for this, not very large, angle value, the stress values reach the yield strength in the large regions of the model.

When the elbow is exposed to both pressure and bending moment, the stress concentration is not much different, as shown in Fig. 8. It can be concluded that the pressurised elbow does not show significantly higher stresses and therefore it would have similar resistance to the bending moment for this configuration and loads. This is in agreement with Robertson et al. (2005), where it is concluded that the collapse moments can even increase with the increase of pressure. This occurs primarily due to the fact that the pressure load prevents/delays the ovalization of the cross section.

In addition, it can be seen that the position with the highest stress values in the case of bending is different than for the internal pressure loading. While the highest stress concentration position in the pressurised elbow is intrados (Fig 5), for the elbow exposed to the bending moment it is the so-called crown position, Figs. 7 and 8.

3.3. Influence of a circumferential defect on bending resistance

The plastic collapse load (in-plane bending moment) of elbows is determined for all the defect dimensions. This load corresponds to the failure of a structure due to the plastic deformation of the ligament in front of the defect.

Dependence of the collapse bending moment on the defect depth is shown in Fig. 9; both opening and closing moments are considered. No significant variation is

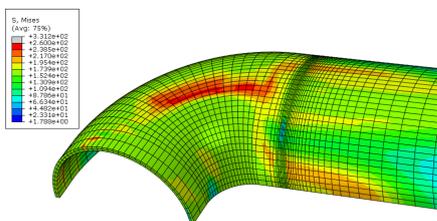


Figure 7. Equivalent von Mises stress field around the notch tip $0.1 \times 2\text{mm}$ – bending angle 2° - without pressure.

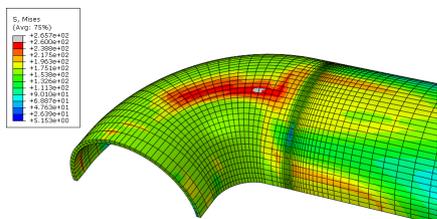


Figure 8. Equivalent von Mises stress field around the notch tip $0.1 \times 2\text{mm}$ – bending angle 2° - with pressure.

obtained for the analysed defect depth and similar conclusions can be drawn for the defect width variation (Fig. 10). It can be seen that the defects discovered by non-destructive examinations do not influence the plastic collapse moment significantly, having in mind that realistic dimensions of the incompletely filled groove are considered. In addition, higher values are obtained for the opening bending moment in comparison to the closing moment, which is in agreement with the most literature findings (e.g. (Hong et al. 2009; J. W. Kim et al. 2008)).

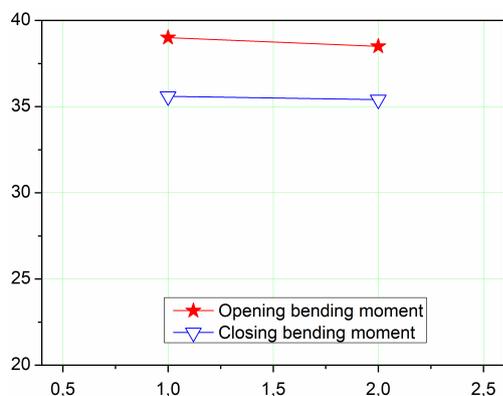


Figure 9. Dependence of the plastic collapse bending moment on defect depth.

Figs. 9 and 10 show that the analysed defect geometries (different depth or width) do not significantly affect the load carrying capacity. This can be explained by the

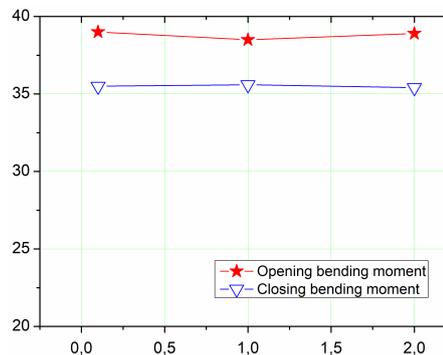


Figure 10. Dependence of the plastic collapse bending moment on defect width.

fact that these defects are not very large in comparison to the pipe wall thickness (up to 1/4 of wall thickness).

Michael et al. (2012) and Michael et al. (2011) also reported a low influence of defects on the elbow load carrying capacity; their work was mainly concerned with the volumetric defects. However, as mentioned previously, the defects considered in Fig. 9 and 10 are not severe - the depth of the notch does not exceed 25% of the pipe wall thickness. In order to assess the pipeline integrity in the presence of the more severe defects, additional models of the elbows are formed. Each of them contains a pre-crack initiated at the bottom of the incompletely filled groove, Fig. 11. The plastic collapse loads, i.e. the bending moments which lead to failure, are shown in Fig. 12. The pre-crack with depth of 6 mm leads to a significant decrease of the collapse load. It is interesting to note that the opening bending moment (which is initially higher) drops more suddenly than the closing one.

In Robertson et al. (2005) and Michael et al. (2012), the range of the defect depths which cause the decrease of the collapse pressure is from 50 to 75% of the pipe wall thickness, which is in agreement with the results obtained in Fig. 12.

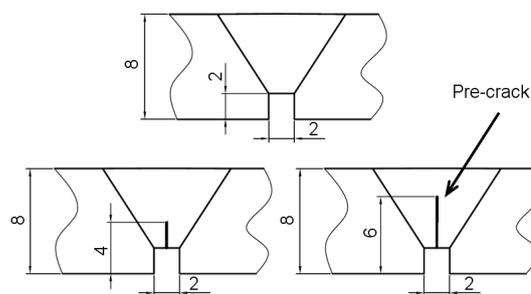


Figure 11. Geometry of the circumferential pre-cracks, from (Dimić et al. 2013).

In Fig. 13, the results obtained from the FE models are compared to the two solutions from the litera-

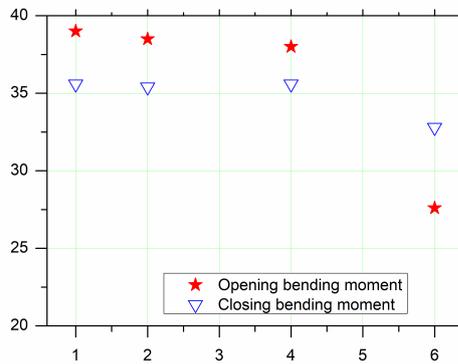


Figure 12. Dependence of the plastic collapse bending moment on the pre-crack depth.

ture, Chattopadhyay et al. (2004) and Chattopadhyay, Nathani, Dutta, and Kushwaha (2000), derived for elbows without defects. It can be seen that the prediction of the opening collapse moment is in agreement with these solutions. However, there is a difference for the closing moment - the FEM results are significantly higher. A possible reason is the fact that boundary conditions are different from those which are typically applied for obtaining the closed-form solutions. As mentioned previously, in this work the boundary conditions correspond to the position of the flange rather close to the elbow, as opposed to the typically analysed configuration with two long pipes attached to both ends of the elbow.

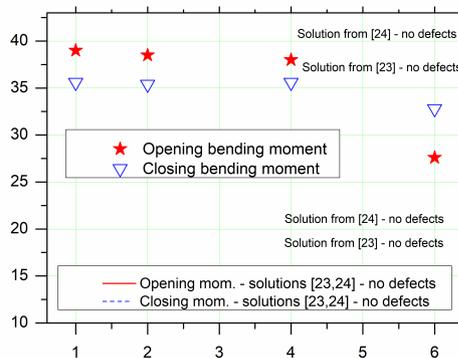


Figure 13. Comparison of the obtained plastic collapse bending moment to some solutions from literature.

Generally, it can be said that the bending of elbows with defects represents a problem, which is not easy to assess in a simplified manner. Namely, the load carrying capacity of an elbow depends on the geometry of the elbow and boundary conditions (due to the attached pipes, flanges, etc.), while the geometry of the defect has a strong influence only if the defect size is large. Additional complexity comes with coupled pressure and bending, which will be reported elsewhere. Generally, when estimating the integrity of an elbow, either by numeri-

cal modelling or some closed-form solutions, one should try to take into account the position of the elbow in the pipeline and joints with other elements as much as possible.

4. CONCLUSIONS

The pressure loading of a pipeline is often accompanied by bending in exploitation, which is especially pronounced for the elbows. – The stress states for the pressure loading, bending loading and combined pressure-bending loading are shown and commented. – The defects discovered by the ultrasonic measurement do not endanger the integrity of the examined pipe elbows subjected to the in-plane bending moment, having in mind that similar plastic collapse moments were obtained for practically all the cases (combinations of circumferential defect depth and width). – For all the defects, opening and closing bending moments were analysed and the values of the collapse moment were greater for the opening bending moment than for the closing one. However, the closing moment is higher for the most severe defects. – Obtained solutions are in good agreement with the literature results Chattopadhyay et al. (2004) and Chattopadhyay et al. (2000) for the opening moment, while much higher closing moments are obtained in this work. A possible cause for this is the elbow configuration, which differs from those most often applied in the literature. The analysis presented here is an initial point for examining the coupled influence of bending and internal pressure, which will be presented in another study.

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