

## ORIGINAL PAPER

# Momentum-heat-mass transfer analogy in gas-solid packed bed at elevated temperatures

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## Abstract

The experimental values of the friction factor ( $f_p$ ) at ambient and elevated temperatures as well as of the heat transfer coefficient ( $h_p$ ) were used to establish the analogy between momentum and heat transfer in gas-solid packed beds of mono-sized spherical glass particles, as  $j_H = f_p/22$  or  $j_H \epsilon = f_p/50$ . Also, the Chilton-Colburn type of momentum-mass transfer analogy was confirmed. These findings are valid for the range of the modified Reynolds number  $Re'_p \approx 20$ -130. The experiments related to  $f_p$  were performed by measuring the pressure drop across the packed bed of particles (0.58, 0.92, 1.04, 1.20, 1.94, 2.98, 3.91, and 4.91 mm diameters) heated to the desired temperature by hot air (temperatures from 20°C to 350°C). The range of gas superficial velocity was from 0.05 to 0.99 m/s, and the bed porosities were from 0.357 to 0.430. The experiments related to  $h_p$  were performed by recording the temperatures of the cold aluminium test spheres (6, 12, and 20 mm in diameter) with embedded K-type (Ni/Al) thermocouples, immersed into the hot packed bed of particles (1.20, 1.94, and 2.98 mm diameters at temperatures from 100 to 300°C) until the thermal equilibrium was reached. The superficial gas velocity and bed porosity varied from 0.30 to 0.79 m/s and from 0.392 to 0.406, respectively. A new correlation for the prediction of the heat transfer factor has been proposed in the form  $j_H \epsilon = 0.30(Re'_p)^{-0.30}$ . The analogies defined in this way leave the possibility of determining the value of the heat and mass transfer coefficients on the basis of the value of the friction factor, which is more common in the literature.

**Keywords:** momentum transfer, heat transfer, mass transfer, packed bed, mono-sized spherical particles, analogy

## 1. INTRODUCTION

In order for the equipment for many unit operations using packed beds of particles in chemical process industry to be adequately designed, it is important that the parameters such as pressure drop of the fluid flow through the bed, heat transfer inside the bed and gas-solid mass transfer be accurately predicted. It seems practical to use the concept of analogies connecting momentum transfer with the heat and/or mass transfer, which was introduced in chemical engineering long ago. Reynolds (1874) formulated the first analogy, which is valid only for turbulent flow. Chilton and Colburn (1934) analogy between mass and heat transfer was similarly defined, both in approach and in its final form showing equality of heat and mass transfer factors,  $j_D$  and  $j_H$ . Particularly, the

Chilton-Colburn analogy was given by empirical correlations, based on the experimental data of turbulent fluid flow through smooth pipes. However, Trinh (2010) made a theoretical derivation of this analogy from first principles followed by the assumption that the index of the power law correlations for the fluid velocity and temperature distributions was the same. Numerous papers have dealt with the Chilton-Colburn analogy in packed beds. Practically it is confirmation of equality of heat and mass transfer factors in packed beds. Mass transfer experiments can be easily conducted in comparison to such complex measurements of the heat transfer. That is why the heat transfer coefficient in packed beds often was not measured but was estimated by analogy with the mass transfer experimental data in packed beds, obtained

whether according to the overall mass transfer phenomena or to the single immersed sphere-to-bed mass transfer phenomena. Bošković-Vragolović, Grbavčić, Janković, and Minić (1996) developed a correlation for  $j_H$  in liquid packed beds of inert particles, based on the experimental data on mass transfer from a single immersed sphere. Motlagh and Hashemabadi (2008) found the Nusselt numbers from the corresponding experimentally obtained Sherwood numbers in the gas-cylindrical particles beds. The same approach was applied in wall affected gas-solid packed beds with orderly stacked cylindrical particles (Mirhashemi & Hashemabadi 2012) and in randomly gas-packed beds of sinter particles (Petrovic & Thodos 1968; Thoenes & Kramers 1958), as well as Dwivedi and Upadhyay (1977), experimentally investigated the overall mass transfer in packed beds and developed correlations for determining mass transfer coefficients based on the mass transfer factor,  $j_D$ . Their results can also be applied to determine the heat transfer coefficient using the analogy of heat and mass transfer in packed beds,  $j_H = j_D$  (Coulson, Richardson, Backhurst, & Harker 1991). Petrovic and Thodos (1968) measured the decrease in the mass of the packed bed over time, because of the evaporation of water and some hydrocarbons from the surface of the bed particles. Based on that, they determined the  $j_D$  factor assuming the ideal gas plug flow through the bed:

$$\varepsilon \cdot j_D = \frac{0.357}{R_p e^{0.395}} \quad (1)$$

Regarding the analogy between momentum and mass transfer in packed beds, the final form proposed is:

$$j_D = \frac{f_p}{20} \quad (2)$$

A large number of correlations of the pressure drop in packed beds of spherical and non-spherical particles have been proposed. Among plenty of overviews of those correlations are those given by (Erdim, Ömer Akgiray, & Demir 2015; Kaludjerovic-Radoicic, Boskovic-Vragolovic, Garic-Grulovic, Djuris, & Grbavcic 2017; Pavlišić, Ceglar, Pohar, & Likozar 2018; Pesic, Kaludjerovic-Radoicic, Boskovic-Vragolovic, Arsenijevic, & Grbavcic 2015; Wang et al. 2017). The most widely used equation for pressure drop in packed beds, the Ergun equation is valid for the incompressible flows through packed beds with neglected wall-to-bed effects and for  $Re'_p$  numbers in the range  $1.2 < Re'_p < 4200$ . This equation is empirical in nature and it is based on the analogy of the fluid flow through the pores of the packed bed with the fluid flow through the tube (Ergun 1952):

$$-\frac{\Delta P}{H} = 150 \cdot \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu}{d_p^2} U + 1.75 \cdot \frac{(1-\varepsilon)}{\varepsilon^3} \frac{\rho}{d_p} U^2 \quad (3)$$

However, research of pressure drop in packed beds at elevated temperatures is sparse (Luckos & Bunt 2011; Pesic et al. 2015; Quintana-Solórzano, Che-Galicia, Trejo-Reyes, Armendáriz-Herrera, & Valente 2018). Luckos and Bunt (2011) found that the values of the pressure drop in a commercial gasifier, calculated according to the Ergun's equation (Ergun 1952), are not satisfactory because of the variation of temperature, composition and pressure of the gases flowing through the packed bed of coal particles in the investigated gasification process. The main conclusion of the study of (Pesic et al. 2015) was that the overall best fit of all experimental data on the friction factor ( $f_p$ ) for air flow through packed beds of spherical glass particles at ambient and elevated temperature is given by the Ergun equation, for  $Re'_p < 500$ . In other words, if it is necessary for one correlation to give the value of the  $f_p$  at the same time both at ambient and at elevated temperatures, then the Ergun correlation can be used for this purpose. However, although the Ergun equation performed best among fourteen correlations tested, the correlation of Reichelt (1972), as well as that of Einfeld and Schnitzlein (2001) gave better results for elevated temperature data only. The majority of the correlations tested showed better fit to experimental data at ambient than elevated temperature (Pesic et al. 2015). Quintana-Solórzano et al. (2018) investigated pressure drop in wall-to-bed effect packed bed of spherical and non-spherical catalyst supports at elevated temperature and pressure, for  $Re_p$  numbers in the range of  $200 < Re_p < 1900$ . Contrary to the results of Pešić, Radoičić, Bošković-Vragolović, Arsenijević, and Grbavčić (2014), it was noted that the mean relative error between experimental and computed data using five correlations (among which was the Ergun correlation and the three correlations that included wall-to-bed effects) did not vary systematically because of temperature changes. They showed an explicit effect of temperature as it was pointed out that pressure drop varied practically linearly with temperature, at constant gas flowrate, with a positive slope whose value rose further with an increase of gas flow rate. This observation was explained by the phenomena of gas expansion during its flow through the packed bed, which is affected proportionally by temperature, as well as by gas viscosity increase with a temperature increase.

In the past, many studies derived the correlations for either heat transfer prediction in packed beds, based on heat transfer factor,  $j_H$ , or Nusselt number, Nu. Pešić et al. (2014) gave the useful overview of the research on the heat transfer in packed beds. They emphasized

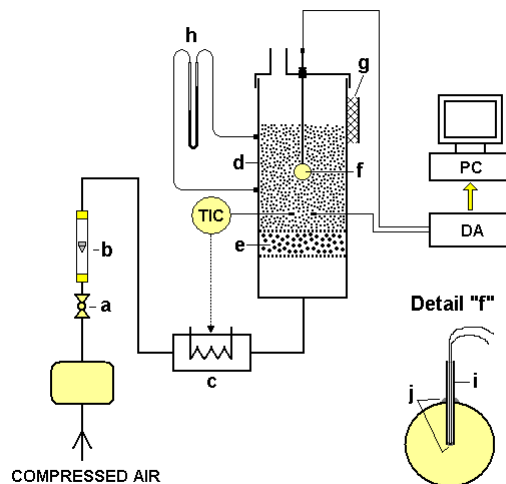
that two approaches are clearly differentiated. The first one is macroscopic approach, which considers the overall heat transfer properties as well as wall-to-bed heat transfer (Balakrishnan & Pei 1979; Handly 1968), and the second one is microscopic approach, which focuses on the individual bed particle (Collier, Hayhurst, Richardson, & Scott 2004; Critoph & Thorpe 1996; Pešić et al. 2014; Scott, Davidson, Dennis, & Hayhurst 2004). Collier et al. (2004) and Scott et al. (2004) proposed similar correlations for heat transfer of the hot sphere immersed in the cold bed of particles, including immersed sphere to particle diameter ratio. Both Nusselt and Reynolds numbers in their correlations are based on the immersed sphere diameter. The correlations proposed by Collier et al. (2004) and Handly (1968) fit experimental data best (Pešić et al. 2014).

The aim of this study was to establish analogy between momentum, heat, and mass transfer in gas-solid packed beds of mono-sized spherical glass particles. The experimental researches on two issues were performed in our previous studies. First, the influence of the bed particle size as well as the size of the immersed sphere on heat transfer between hot beds (from 100°C to 300°C) to cold immersed sphere was experimentally determined by investigating the heat transfer coefficient,  $h_p$  (Pešić et al. 2014). Second, the influence of gas temperature on the packed bed pressure drop by investigating the friction factor  $f_p$ , for gas flow through beds at ambient and elevated temperatures (from 20°C to 350°C), was experimentally conducted (Pesić et al. 2015). An attempt was made to establish relations between  $f_p$ ,  $j_H$ , and  $j_D$ , based on experimental data for  $f_p$  not only at ambient but also at elevated temperature, which is not the case in the literature so far.

## 2. EXPERIMENTAL

In this study, experimental results from the studies by Pesić et al. (2015); Pešić et al. (2014) were used for establishing analogies. The experiments of the bed pressure drop at room temperature were performed using a Plexiglas cylindrical column of a 62 mm diameter and 300 mm height. For the experiments of the bed pressure drop both at room and elevated temperature, as well as for heat transfer experiments, the thermally insulated packed bed column of a 119 mm diameter and 301 mm height was used. This column is presented schematically in Figure 1. The column (d) was equipped with a distributor and the calming section (e), in order to ensure the uniform flow of air through the packed bed. The airflow was introduced at the bottom of the column using a compressor and heated by an electric heater (c) connected to the temperature controller allowing for bed temperature regulation (TIC). The temperature was measured at the bottom

and at the top of the column, showing the difference less than 5°C, so the effects of the temperature variation along the bed height were neglected.



**Figure 1.** Experimental system: (a) Valve, (b) Rotameter, (c) Electric heater, (d) Column, (e) Distributor, (f) Test sphere, (g) Thermal insulation, (h) Manometer, (i) steel sheath, (j) Brazing points

The mean value of the temperature was then calculated and used for the calculation of thermo-physical properties of air (density and viscosity). The pressure drop through the packed bed was measured using a water manometer (f). The height between the pressure taps (h) was 200 mm. The measurements were performed for different particle diameters, air velocities, and bed temperatures. Bed particle Reynolds number  $Re_p$  varied between 2 and 503. The bed geometric aspect ratio ( $D_c/d_p$ ) in the experiments was between 12.6 and 108.1. The wall effects on pressure drop were not examined experimentally. Regarding this issue, the literature discussing the bed geometric aspect ratio values for which the wall effects are negligible is somewhat divided. Generally, at the bed geometric aspect ratio less than 10, the wall effects are considered negligible, but some papers show wall effects to be significant at the ratios as high as 15-20 (Eisfeld & Schnitzlein 2001; Felice & Gibilaro 2004). In Pesić et al. (2015) it was assessed based on the work of Reichelt (1972) that for the bed geometric aspect ratio of 12.6 the wall effects are quite small even for low values of  $Re'_p$ , i.e. they can be considered as negligible for the experimental conditions used. In order for the temperature of the test sphere to be measured, after adjusting the hot air flow rate manually by the valve (a) and rotameter (b) and reaching the desired stable value of the bulk temperature, the aluminum test sphere initially at room temperature was immersed into the bed above the gas distribu-

**Table 1.** Particles and fluid characteristics.

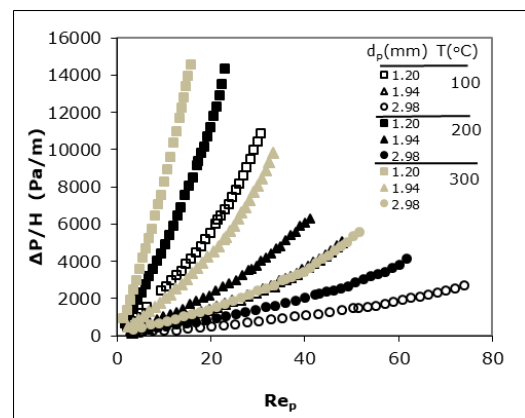
No.	$d_p$ mm	$\rho_p$ kg/m <sup>3</sup>	$d_s$ mm	$\rho_s$ kg/m <sup>3</sup>	$C_{ps}$ J/kgK	$\lambda_s$ W/mK	$\epsilon$
<b>Pressure drop experiments</b>							
1	0.58	2679					0.357-0.383
2	1.04	2809					0.400-0.421
3	1.20	2661					0.376-0.401
4	1.94	2515					0.363-0.424
5	2.96	2533					0.374-0.422
6	3.91	2555					0.376-0.426
7	4.91	2555					0.385-0.430
<b>Heat transfer experiments</b>							
8			6				
9	2.98	2509	12				0.406
10			20				
11			6				
12	1.94	2507	12	2670	910	215	0.406
13			20				
14			6				
15	1.20	2641	12				0.392
16			20				
<b>Air thermo-physical characteristics</b>							
$\rho = 1,20163 - 3,17322 \cdot 10^{-3} \cdot T + 4,83985 \cdot 10^{-6} \cdot T^2 - 2,97565 \cdot 10^{-9} \cdot T^3$							
$\mu \cdot 10^6 = 17,05643 + 0,047641 \cdot T - 2,6739 \cdot 10^{-5} \cdot T^2 + 1,05249 \cdot 10^{-8} \cdot T^3$							
$\lambda = 0,0244 + 8 \cdot 10^{-5} \cdot T - 10^{-8} \cdot T^2 - 2 \cdot 10^{-11} \cdot T^3$							

tor. The K-type (Ni/Al) thermocouple was inserted into the center of the test sphere through the drilled hole (f) and connected to the data acquisition system, which registered the temperature of the test sphere over time with sampling frequency of 1 Hz. Experimental measurements were performed by varying the bed particles diameter, the test sphere diameter, gas temperature and gas velocity. Bed particle Reynolds number  $Re_p$  varied between 11 and 70. Particles and fluid characteristics are summarized in Table 1, and experimental conditions are summarized in Table 2. The effects of glass dilatation were neglected in the calculations, as the volume change of glass is less than 1% for the temperature raise of 300°C. Note that for each experimental run minimum fluidization velocity was calculated taking into account the corresponding gas density and viscosity by using the Wen and Yu (1966).

### 3. RESULTS AND DISCUSSION

Aimed to compare the pressure drop related to different particle diameter, Figure 2 shows the dependence of the pressure drop gradient on  $Re_p$  for three used particles accounting for three sets of experiments performed at 100, 200, and 300°C. For particles of 1.20 mm, 1.94 mm, and

2.98 mm values of the bed porosity were 0.396, 0.410, and 0.411 respectively.



**Figure 2.** Pressure drop gradient versus particle Reynolds number for different particles at elevated temperatures.

As expected from the typical behavior, pressure drop gradient increases nonlinearly when  $Re_p$  increases. From the corresponding curve slopes, it can be concluded that such an increase became more pronounced as bed temperature was raised, for all particle diameters. At a specific value of  $Re_p$  and temperature, the pressure drop gradient increased as particle size decreased. Particularly, based

**Table 2.** Range of the experimental conditions.

No.	Number of exp. runs	D <sub>c</sub>	T	T <sub>s0</sub>	U	U/Umf
		mm	°C	°C	m/s	
<b>Pressure drop experiments</b>						
1	62	20			0.06-0.23	0.16-0.96
2	62	20			0.03-0.52	0.07-0.82
3	119	20, 100, 200, 250, 300, 350			0.03-0.78	0.06-0.96
4	28	119	20, 100, 200, 250, 300, 350		0.03-0.99	0.04-0.86
5	119	20, 100, 200, 250, 300, 350			0.05-0.98	0.03-0.54
6	62	20			0.09-0.98	0.02-0.27
7	62	20			0.11-0.87	0.02-0.24
<b>Heat transfer experiments</b>						
8	35	103-291		19-35	0.30-0.76	0.20-0.44
9	20	99-306		19-33	0.31-0.79	0.20-0.46
10	20	91-323		19-34	0.29-0.79	0.19-0.46
11	32	106-297		21-27	0.31-0.70	0.28-0.62
12	20	119	109-320	19-23	0.31-0.79	0.28-0.69
13	20		99-311	19-27	0.31-0.79	0.28-0.69
14	22		112-302	19-23	0.32-0.56	0.47-0.83
15	17		99-303	20-28	0.31-0.63	0.45-0.94
16	16		106-314	23-29	0.31-0.63	0.45-0.95

on the experiments at specific conditions of  $Re_p$  and the temperature of 100°C, the particles of 2.98 mm caused about a ten times lower pressure drop gradient compared to particles of 1.20 mm and about three times lower pressure drop gradient compared to particles of 1.94 mm. Figure 2 shows very similar trends for temperatures of 200°C and 300°C. From experimental data of pressure drop,  $f_p$  was calculated using Eq. 4 for friction factor based on the Ergun equation:

$$f_p = \left( -\frac{\Delta P}{H} \right) \cdot \frac{d_p}{\rho U^2} \cdot \frac{\varepsilon^3}{1 - \varepsilon} \quad (4)$$

Figure 3 shows the dependence of friction factor on the modified Reynolds number ( $Re'_p$ ), for three used particles at ambient and elevated temperatures. It can be noticed that there is a certain influence of temperature on the measured values of  $f_p$  in the case of particles with a diameter of 1.20 mm, while in the case of larger particles with a diameter of 1.94 mm and 2.98 mm, this influence becomes smaller. At room temperature, (20°C)  $f_p$  value is the highest, and decreases with increase in temperature. The difference between the values of  $f_p$  at different temperatures decreases as the value of  $d_p$  increases.

The largest differences were observed when temperature increased from 20°C to 100°C, while a further increase in temperature had a smaller effect on variation of  $f_p$ . In our previous work, it was shown that the Ergun correlation adequately represents the experimental results

obtained at room temperature while at elevated temperatures there was some deviation of the calculated from the measured values (Pešić et al. 2014). This deviation was more noticeable for smaller particles and decreases with increase in bed particle size, as can be seen in Figure 3. Those observations could be a good base for further investigations of pressure drop in porous media under elevated temperature, in order to resolve the influence of elevated temperature on these phenomena. Although the Ergun equation, in general, predicts pressure drop values with certain accuracy, it has been subject of some objections related to the particle shape and size, bed geometric aspect ratio and  $Re_p$  number values, so many researches derive new values of constants in this correlation, as well as developing new correlations. Certain, the note given in numerous papers dealing with the applicability of the Ergun equation can be considered valid for packed bed pressure drop at elevated temperatures as well. This note says that one should take into account when it comes to coefficients in the Ergun equation that the coefficients values are specifically appropriate for conditions identical or similar to the experimental conditions in which these coefficients were obtained. The heat transfer coefficient,  $h_p$ , can be determined from the heat balance equation for the cold immersed test sphere in the packed bed, during the heating of such teste sphere by the hot flowing gas. This method is graphically in its basis, since  $h_p$  is to be calculated from the slope of the experimentally obtained plot of  $\ln[(T - T_s)/(T - T_{s0})]$  against time, which is linear.



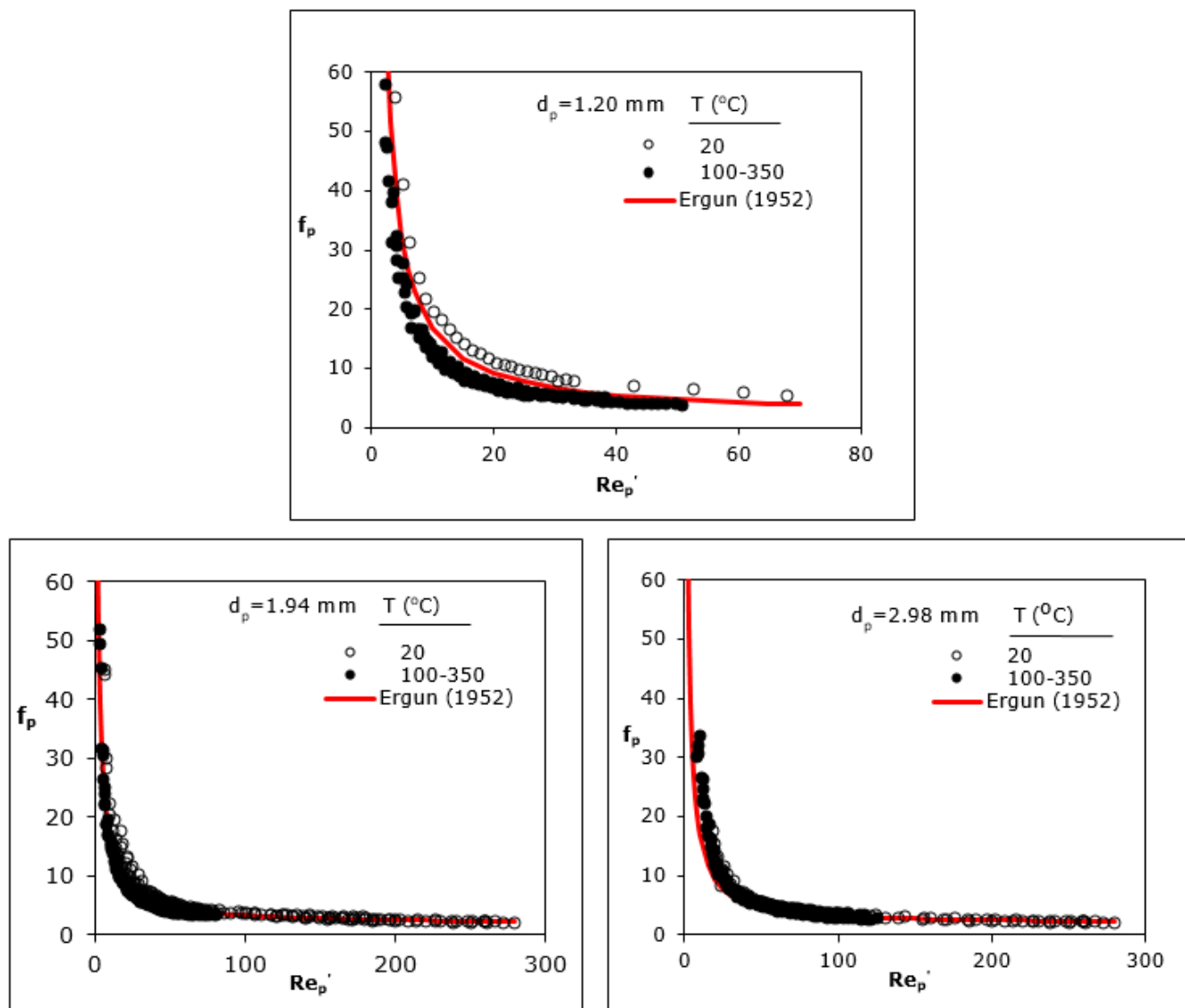


Figure 3. Dependence of  $f_p$  on  $Re_p'$  at ambient and elevated temperatures.

As it was shown in our previous work, the assumptions of negligible resistance to conductive heat transfer inside the test sphere, as well as negligible radiation heat transfer for temperatures below 600°C, were satisfied (Pešić et al. 2014). In Figure 4 the dependences of the experimental values of  $h_p$  on superficial gas velocity  $U$  for all experimental conditions, are given as a dependence of  $Nu$  ( $h_p d_p / \lambda$ ) on  $Re_p$ .

The increase of  $h_p$  with the increase in  $U$  is noticeable, primarily because of increased contact between the immersed test sphere and the surrounding bed. This is consistent with the findings of Collier et al. (2004) and Scott et al. (2004). Further, the general trend that  $h_p$  is the highest for the smallest bed particles and the lowest for the largest bed particles can be seen, although it is not so significantly expressed, having in mind that the  $Nu$  number increases with the increase in  $h_p$ , as well as with the in-

crease in  $d_p$ . The lack of clearly distinguished influence of bed particle size on the  $Nu$  value can be explained by the fact that the porosities of the bed of three different particles are of very close values. Since the experiments were performed independently for each immersed test sphere, there were no continuous and systematic changes of the particulate system structure in the investigated range of porosities. Pešić et al. (2014) emphasized that regardless of the fact that the bed particle size does not affect significantly the heat transfer, it can be concluded with certainty that  $h_p$  depends on the ratio ( $D_s/d_p$ ). Since the value of  $h_p$  is sensitive to the value of  $D_s$ , it is a sign that both convective heat transfer from the flowing gas to the immersed test sphere and heat transfer by conducting from bed particles to the immersed test sphere are present. Scott et al. (2004) also showed this behavior of packed gas-solid beds. The heat transfer factor values,  $j_H$ , were calculated

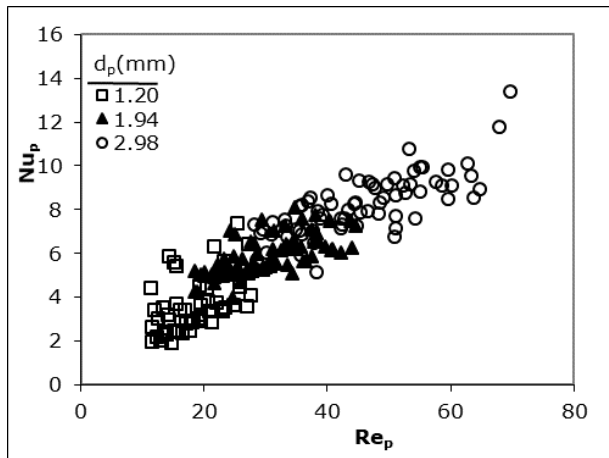


Figure 4. Dependence of  $Nu$  on  $Re_p$  for different particles.

from Nusselt numbers, as  $Nu/(Re_p Pr^{1/3})$ . Figure 5 shows the dependence of the  $j_H \epsilon$  on the  $Re'_p$  for all-experimental data together with the best-fit line given by the following equation (mean deviation is 15.8%):

$$j_H \epsilon = 0.30(Re'_p)^{-0.30} \quad (5)$$

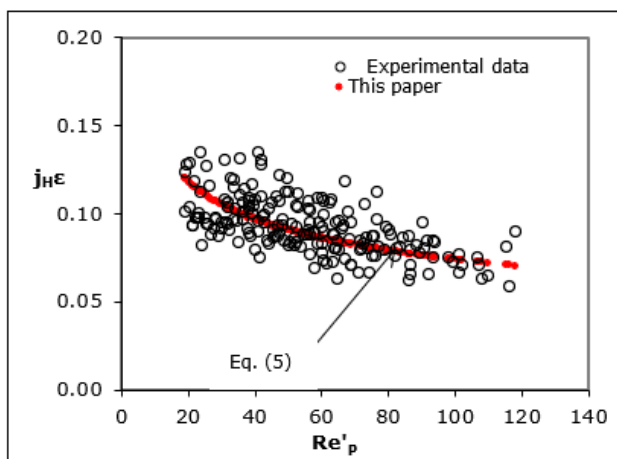


Figure 5. The heat transfer factor as a function of the  $Re'_p$  for all experimental data.

Pešić et al. (2014) give the comparison of the experimental data and the literature correlations. Those correlations are given either in the form of  $Nu$  number or as dependence  $j_H = f(Re_p)$ . Eq. 5 can be considered as one possible way to correlate experimental data of heat transfer in packed beds, since it is arranged to include differently conceived dimensionless groups, heat transfer factor in the form of a dimensionless group  $j_H \epsilon$  and a modified Reynolds number  $Re'_p$ . This has been done because in order for the correlation for the heat transfer in the packed bed to be able to describe the phenomena in the best possible way it must allow for porosity effects. From Figure

5 it can be seen that the inclusion of porosity in the dimensionless group does not unify the data. The reason is that the experiments were performed in packed beds with the porosities of very close values ( $\approx 0.40$ ). In other words, all packed beds were normally packed beds, i.e. there was no densely packed bed and/or loosely packed bed, in which case the inclusion of the porosity parameter in the dimensionless group of the heat transfer factor would certainly equalize data values for the whole range of experimental conditions. The correlation, as is given in Figure 5, could play a role in establishing an analogy between heat transfer and mass and momentum transfer in a gas-solid packed bed at ambient and elevated temperature. The first step in establishing the analogy of the momentum transfer and heat transfer was to compare the experimentally obtained data for  $j_H$  and  $f_p$ , as is given in Figure 6.

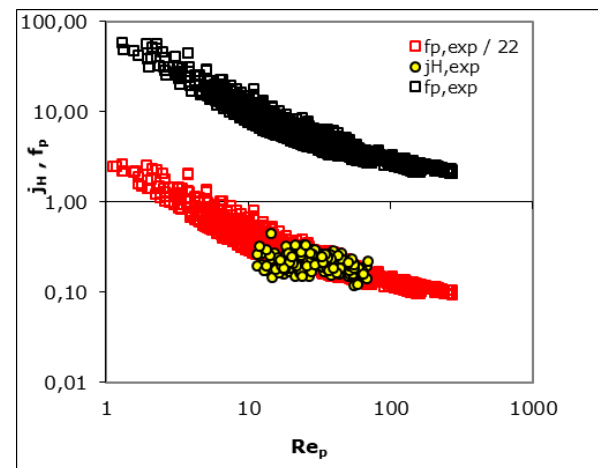


Figure 6. Comparison of experimental data for  $j_H$  and for  $f_p$  at ambient and elevated temperature.

As can be seen from Figure 6, the range of the  $Re_p$  number used in experiments for  $f_p$  was much larger than the range of the  $Re_p$  number in experiments for  $h_p$  ( $j_H$ ), where the  $Re_p$  number was in the range from  $\approx 10$  to 80. The resulting analogy of momentum and heat transfer, in the form  $j_H = f(f_p)$ , is:

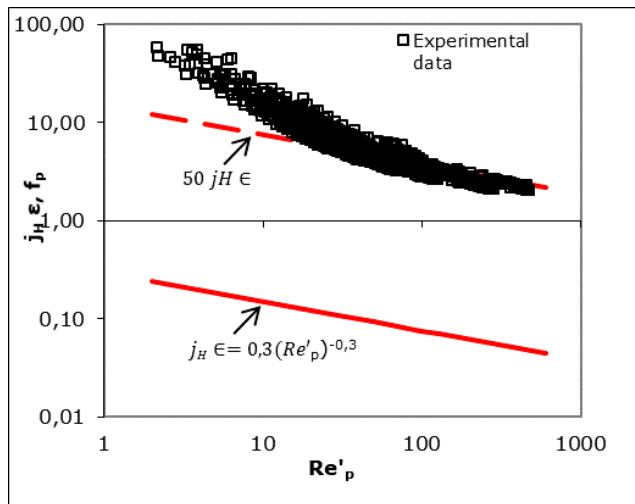
$$j_H = \frac{f_p}{22} \quad (6)$$

This equation is very similar to Eq. 2. Figure 7 shows the dependences of the  $f_p$  and  $j_H \square$  on the  $Re'_p$ . Actually, a rearranged Eq. 6 is given as a best fit of experimental data for momentum and heat transfer in the investigated packed bed:

$$j_H \epsilon = \frac{f_p}{50} \quad (7)$$

From Figures 6 and 7, it can be seen that the validity of Eq. 6 and Eq. 7 exists for a range of the  $Re_p$  num-

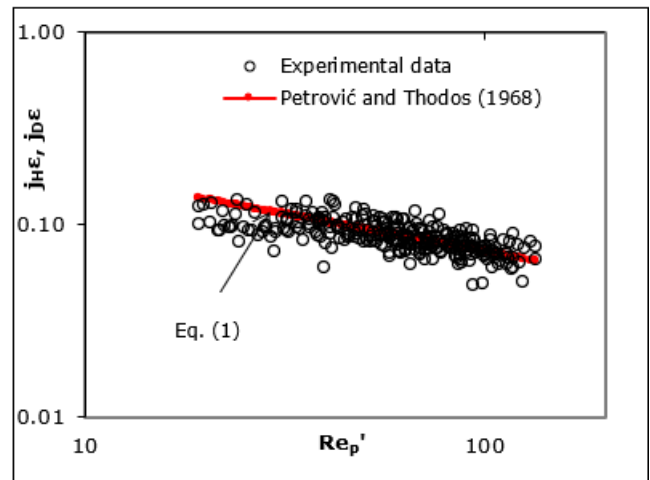
ber from  $\approx 10$  to 80, while for smaller values of the  $Re_p$  number there is a large deviation of Eq. 6 from the experimental results for the  $f_p$ .



**Figure 7.** Analogy of momentum transfer at ambient and elevated temperature and heat transfer.

Regardless of the form of the dependencies shown in Figures 6 and 7, the conclusions that can be drawn from these dependencies are identical. In the examined range of  $Re_p$  numbers, values of the  $f_p$  are significantly higher than  $j_H$  values due to the fact that the momentum transfer, in addition to the friction between the gas and the surface of the particles, is largely influenced by the effects of drag resistance, local flow resistances, local turbulence, as well as local changes of fluid kinetic energy in the capillaries of the packed bed. The initial setting of the Chilton-Colburn analogy is based on the analogies of the transfer of momentum, heat, and mass only if the transfer of momentum is affected exclusively by friction (e.g., flow through a pipe). The flow through the packed bed at elevated temperatures is very complex and there are a large number of factors that affect fluid energy losses during fluid flow through such a particulate system, so all of these factors are shown by the value of the  $f_p$  coefficient. However, in the scope of the experimental results given in this paper, the trends of the values of  $j_H$  (or  $j_H \epsilon$ ) and  $f_p$  are the same and their mutual connection can be expressed by analogy equations, Eq. 6 and Eq. 7, which leave the possibility of determining at least the approximate value of the heat transfer coefficient on the basis of the value of  $f_p$ , which is more available in the literature sources. Mass transfer during gas flow through packed beds has not been experimentally determined, so Eq. 1 (Petrović & Thodos 1968) was used to establish an analogy with heat transfer. Experimental results of heat transfer showed a complete analogy with mass transfer in packed beds (Figure 8):

$$j_H \epsilon = j_D \epsilon \quad (8)$$



**Figure 8.** Analogy of heat and mass transfer.

To our best knowledge, so far there has been no relation in the literature between  $f_p$ ,  $j_H$ , and  $j_D$ , based on experimental data for  $f_p$  at elevated temperature. In this regard, the obtained results provide a basis for further research of the analogy of the transfer phenomena in a packed gas-solid bed at elevated temperatures, for wider ranges of the  $Re_p$  number.

#### 4. CONCLUSIONS

Within this work, experimental results of the pressure drop in a packed gas (air)-solid bed at room temperature and at elevated temperatures (100-350°C), given at Pešić et al. (2015), are analyzed. Also, the heat transfer coefficients from the hot packed bed to the cold immersed fixed sphere, which were determined experimentally by Pešić et al. (2014), are commented. There was a certain influence of temperature on the measured values of the friction factor ( $f_p$ ) for the smallest bed particles, while for the larger particles this influence became much less evidenced. At room temperature (20°C) the  $f_p$  value was the highest, and decreased with the increase in temperature. The largest differences were observed when temperature increased from 20°C to 100°C, while a further increase in temperature had a smaller effect on the variation of  $f_p$ . Based on the experimental data, the new correlation for prediction of the heat transfer factor in the gas-solid packed bed of inert spherical particles has been proposed in the form  $j_H \epsilon = 0.30 (Re'_p)^{-0.30}$ . For the range of experimental tests analyzed in this paper ( $Re_p \approx 10-80$ , i.e.  $Re'_p \approx 20-130$ ), the trends in the values of  $j_H$  (or  $j_H \epsilon$ ) and  $f_p$  were the same, with significantly higher values of the  $f_p$ . Their mutual connection has been expressed by



momentum and heat transfer analogy equations  $j_H = f_p/22$  and  $j_H \epsilon = f_p/50$ . In addition, for the gas flow through the packed bed, an analogy of heat and mass transfer was set as  $j_H \epsilon = j_D \epsilon$ , using the Petrovic-Thodos correlation (Petrovic & Thodos 1968) for mass transfer in packed beds. The obtained results provide a basis for further research of the analogy of the transfer phenomena in a packed gas-solid bed at elevated temperatures, for wider ranges of the  $Re_p$  number.

## NOMENCLATURE

- $C_p$  – Gas heat capacity (J/kgK)
- $C_{ps}$  – Heat capacity of the test sphere (J/kgK)
- $D_c$  – Column diameter (m)
- $d_p$  – Particle diameter (m)
- $d_s$  – Test sphere diameter (m)
- $f_p$  – Friction factor (-)
- $H$  – Bed height (m)
- $h_p$  – Heat transfer coefficient (W/m<sup>2</sup>K)
- $j_H$  –  $(Nu/Re_p Pr^{1/3})$  Heat transfer factor (-)
- $j_D$  –  $(Sh/Re_p Sc^{1/3})$  Mass transfer factor (-)
- $Nu$  –  $(h_p d_p/\lambda)$  Nusselt number (-)
- $Pr$  –  $(\mu C_p/\lambda)$  Prandtl number (-)
- $Re_p$  –  $(\rho U d_p/\mu)$  Particle Reynolds number (-)
- $Re'_p$  –  $Re_p/(1-\epsilon)$  Modified Reynolds number (-)
- $T$  – Gas temperature (°C)
- $T_s$  – Test sphere temperature (°C)
- $T_{s0}$  – Initial test sphere temperature (°C)
- $U$  – Gas superficial velocity (m/s)
- $U_{mf}$  – Minimum fluidization velocity (m/s)

## Greek letters

- $\Delta P$  – Pressure drop (Pa)
- $\epsilon$  – Bed porosity (-)
- $\lambda$  – Gas thermal conductivity (W/mK)
- $\lambda_s$  – Test sphere thermal conductivity (W/mK)
- $\mu$  – Gas viscosity (Pas)
- $\rho$  – Gas density (kg/m<sup>3</sup>)
- $\rho_p$  – Particle density (kg/m<sup>3</sup>)
- $\rho_s$  – Test sphere density (kg/m<sup>3</sup>)

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